Dynamical Evolution of a Collinear Planetary System

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Abstract

Starting from Talbott, several mythologists have argued that during the initial phase of human history the solar system configuration was radically different than the present one, the planets being aligned with the Sun during their revolution, with Saturn in a fixed dominating position in the sky (the polar model) and Jupiter being not visible. In a previous paper the equations defining the equilibrium configuration of the aligned system were considered and numerically solved. The equations define an underdetermined nonlinear homogeneous system, whose solution can be expressed in term of one scalar parameter α (the distance from the Sun to the Earth in astronomical units). Analysis of the solution is in basic agreement with the tenets of the polar model. In this paper we solve numerically the initial value differential equations defining the evolution of the considered planetary system under the standard assumptions on the forces acting on a planetary system. The system is solved for different values of the parameter α, the main aim being a numerical evaluation of the stability of the considered aligned configuration. We consider the system to have lost alignement when the angle between the position vectors of Earth and Saturn exceeds 10%. We find that for small values of α loss of alignement is fast, just a matter of days; the needed time for the loss increases to a maximum of about 2.5 months at $\alpha \cong 0.8$; then it decreases and for large α seems to stabilize at about one month. The obtained results therefore do not confirm the dynamical stability of the polar model. The nonlinear dependence of the "destabilization" time from α is an interesting unexpected result and may point to a till now undetected existence of stability intervals for α. In order to possibly reestablish the dynamical feasibility of the polar model one should introduce in the physical model forces not here considered, e.g. tidal forces and/or electromagnetical forces.

1. Introduction

Analysis of the invariant elements in religions and in mythologies worldwide has led several scholars, e.g. Talbott (The Saturn Myth, 1980), to introduce the idea that during a remote time in the human experience the solar system was radically different than now. Such ancient configuration, called the polar model, some of whose features are recalled below, evolved according to Talbott et al. into the present state after probably violent and dramatic events during human memory. Some basic elements of the polar model are the following:

- The Sun was not visible (we understand this as unvisibility from the part of the Earth where man was living).
- Saturn was the dominating object in the sky. It loomed large and his position in the sky appeared fixed.
- Venus appeared centered in the middle of Saturn; it was often referred to as the eye of Saturn.
- Mars appeared centered in the middle of Venus, but with variable angular size; it was often referred to as the pupil of the eye.
- No other planets, in particular not Jupiter, were visible.

The above features suggest the following configuration of the solar system:

- A synchronous revolution of the visible planets, including the Earth, possibly along circular orbits, with the exception of Mars.
- The fixed position of Saturn can be best explained if it is assumed that the Earth revolved keeping the same face towards the Sun (as the Moon does now with respect to the Earth). The Earth's axes of revolution and of rotation would coincide and would be orthogonal to the ecliptic plane. The "day" of the Earth would be equal to the year.
- Nonvisibility of the Sun can be explained assuming that living conditions on the terrestrial hemisphere facing the Sun were impossible for man, either because that emisphere was too hot or was all occupied by ocean. Hence man lived on the less hot hemisphere facing the external planets, illuminated by the light of the Sun as reflected by the external planets in a fixed position.

- Nonvisibility of Jupiter can be explained either by its irrelevance, being much farther away, or by a location in alignement behind Saturn or the Sun, or finally by its absence. It is tempting to hypothesize that Jupiter has been recently captured, possibly from the huge molecular cloud located in the Orion Gould belt direction, that was crossed by the solar system about ten million years ago. The capture of Jupiter, presumibly with a lot of satellites, must have been a truly catastrophical event for an aligned solar system, leading to a collapse of the configuration. The planetary explosion argued for by T. Van Flandern (Dark Matter, Missing Planets and New Comets, 1999), who dates it at about 3.2 million years ago and sees it as the source of many present features of the solar system, may be explained in this context, where it may have actually been the main factor allowing the capture of Jupiter.

2. Gravitational equilibrium features of the polar model

The equilibrium equations governing a system of m aligned planets were given and solved in Spedicato and Del Popolo (1998, Equilibrium distances of a collinear planetary system, Report DMSIA 98/13, University of Bergamo, 1998) and Spedicato (Aeon V,4, 1999) under the following assumptions:

- the planets are aligned and have the same angular velocity
- the orbits are circular
- the only relevant forces are the gravitational forces; the masses are considered concentrated in their baricenter
- the planets are spheres with the same mass and size as now
- the planets alignment order is given
- no satellites are considered.

Let us consider a system of m planets revolving around a certain star. Let the masses of the m planets be $M_1, \ldots, M_m$ and let M_0 be the mass of the star. Let all these bodies be revolving around their baricenter on circular orbits, with the same period and lying on a same line. Such fixed geometry configuration is feasible for the m-body pure gravitational problem and is stable under certain conditions on the masses of the bodies and the orbital parameters, see for instance Milani and Nobili (Celestial Mechanics 31, 213-240, 1983). Let the distances of the considered planetary masses from the baricenter be $x_1, \ldots, x_m$. Let V_i be the modulus of the velocity of the ith planet. Then syncronicity and circularity of the orbits imply for every $i,\ j$:

$$V_i/x_i = V_j/x_j. \tag{1}$$

Under the given assumptions the gravitational force acting on the ith planet must be equal to the centripetal force, hence we obtain, with x_0 the distance of the star from the baricenter and G the gravitational constant, for $i = 1, \ldots, m$

$$GM_i \left[\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2 \right] = M_i V_i^2/x_i. \tag{2}$$

Cancelling M_i in (2) and using (1) to express V_i^2 in term of, say, the Earth velocity V_E and distance x_E we obtain, for $i = 1, \ldots, m$

$$G \left[\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2 \right] = x_i V_E^2/x_E^2. \tag{3}$$

Equations (3) are a system of m equations in the $m+2$ variables x_i and V_E. We can get V_E from one of the equations (3), for instance from the Earth's equation, obtaining a system of $m-1$ equations for the $m+1$ variables x_i. Such equations have the following form, with E the index of the Earth in the given planetary sequence

$$\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2 = x_i/x_E [\sum_{j=0}^{E-1} M_J/(x_E - x_j)^2 - \sum_{j=E+1}^{m} M_j/(x_E - x_j)^2]. \tag{4}$$

Notice that G does not appear in (4) and that if we write the above system as $F(x) = 0$, then $F(\alpha x) = F(x)/\alpha^2$, hence the system is a homogeneous one of degree -2. Therefore if x is a solution, so is αx for any nonzero α. Finally the equation defining the baricenter is given, being homogeneous of degree $+1$

$$\sum_{j=0}^{m} x_j M_j = 0 \tag{5}$$

It may be observed that if we add to (2) the equation related to the star, then equation (5) becomes a consequence of (1) and (2), a fact essentially stating that the system behaves like a rigid body.

The above given equations define an underdetermined system of m equations for the $m+1$ distances of the star and the planets from their baricenter. They have been solved numerically for three planetary configurations related to the solar system, having assumed as unit of mass the Earth mass, as unit of length the astronomical unit (i.e. the distance Earth to Sun, about 149.600.000 km) and as unit of time the year.

The numerical solution was obtained as follows. The value of $\alpha = x_E$ was set to one, thereby obtaining a determined nonlinear system. The system was solved by a MAPLE code based upon Newton method, from different starting points, all giving the same solution vector x^*. For different values of α the solution can be obtained by multiplying by α the vector x^*, in view of the homogeneity of the system.

Table 1 gives the numerical results for three considered configurations A,B,C. Configuration A consists of the planets Earth, Mars, Venus and Saturn in this order; configuration B has the Moon between Earth and Sun; configuration C has the Moon between Mars and Earth.

Table 1

Body	A	B	C
Sun	$-3.0035E_4$	$-3.0030E-4$	$-1.355E-5$
Earth	1	1	1
Moon		0.99328	1.003925
Mars	1.00519	1.00517	1.005444
Venus	1.00988	1.00984	1.010039
Saturn	1.0522	1.015211	1.05228

Remark 1. For case C the total tidal forces of the external bodies (Moon, Mars, Venus and Saturn) over the Earth are computed to be 7.97 times greater than the present tidal forces due to the Sun. For case B the total tidal forces are 7.73 times greater than the present tidal force due to the Sun. Hence the tidal forces in the considerd configuration are mainly given not by the Moon, as is the case now, but by the other planets.

Table 2 gives, under the headings α-planet the angles in degrees under which Mars, Venus, Saturn and Moon would be viewed from the Earth, assumed to be at one astronomical unit from the baricenter.

Table 2

α-Mars	α-Venus	α-Saturn	α-Moon
0.5894	0.5699	1.0899	0.4186

Remark 2. From the equilibrium equations one can compute the modulus of the velocity of the considerd bodies in their circular orbits. For the Earth, assumed at one A.U. from the baricenter, the speed is about 5.4 A.U., hence about 20% less than now (being now 2π), or, in km/sec, about 24 km/sec.

The following observations follow from an inspection of the Tables:

- in agreement with the Talbott *et al.* interpretation of the mythological record planet Saturn revolves quite close to the Earth. Its distance from the Sun is only about 5% greater than the Earth distance, if Jupiter is not considered, about 4.5% if Jupiter is considered. Jupiter would be only about 12% more distant from Sun than Earth and completely invisible, its visibility angle being smaller than the angles of Mars, Venus and Saturn

- Venus visibility angle is about the half of the Saturn angle, this value being essentially unaffected by the presence of Jupiter

- Mars visibility angle appears to be about 7% greater than the Venus angle; thus Venus would not have been visible, falsifying a tenet of the polar model. However there is a natural escape from this conclusion. We have used the present diameters of the planets. If Venus had in the previous configuration a sufficiently larger atmospheric mass, that was lost in the catastrophical events that ended that configuration (we suggest a grazing impact with the newly arrived Jupiter), then its diameter would have been larger and the problem would disappear

- Saturn visibility angle would be greater than the present Moon's angle for x_E less than about 1.5 astronomical units

- gravitational tidal force of the external planets would be larger than the Sun's tidal force, the planets closest to the Earth having the greatest force. Notice that the ratio of these forces is independent of the actual Earth distance from Sun, due to the homogeneity property of equation (4). Notice also that the ratio of the planetary tidal forces over the Sun's tidal force would be comparable with the present ratio of Moon's over Sun's tidal force (about 2.2), but the resulting action would be greater if the Earth kept the same face towards the planets. This fact may have had remarkable influence on the geological structure of the Earth (it is tempting to associate the formation of a unique continental mass, the Pangea, with such a tidal action).

- since the numerical results assign to Saturn an orbit much closer to the Sun than presently, it is quite possible that the atmospheres of Saturn and likely of Venus were hotter than presently and consequently more expanded, resulting in a visible diameter of the planets larger than the present diameter, which has been considered in the computations.

3. Dynamical evolution of the planetary aligned system

The equilibrium positions obtained by solving the relevant equations are basically in agreement with the main features of the polar model. In order to have a full validation of the model two more questions should be addressed:

1. how the aligned configuration originated
2. if the aligned configuration, with the considered planets, is dynamically stable (recall from Milani and Nobili (1983) that stability of an aligned configuration depends on relative sizes of masses and some orbital parameters)

In this paper we will not address question (1), whose solution is not obvious, but might be related to the new theory for birth of planets and satellites proposed by Van Flandern (1999, op.cit.), namely emission of such bodies by a fast rotating parent body (star for the birth of planets, planet for satellites), in opposite, and hence at least initially, aligned directions. Here we will deal with the stability question by numerically integrating the motion equations of the considered system and determining how long it takes for the aligned configuration to be unravelled.

Assuming that the planets are homogeneous spheres and disregarding tidal effects, the dynamics of the considered system is equivalent to the dynamics of $m+1$ points, associated to the Sun and to the m planets. If $x_i(t)$ is the position at time t of the i-th body in a suitable reference system, then the motion of the i-th body is described by the classical Newton equation, where $x(t)$ is the vector containing the positions at time t of all the considered bodies

$$M_i \ddot{x}_i(t) = F_i(x(t), t) \tag{6}$$

Assuming that the bodies move under only the gravitation force, depending on the instantaneous position of the bodies at time t (hence disregarding delay effects), then we have

$$F_i(x(t),t) = F_i(x(t)) = -M_i G \sum_{k\neq i} M_k e_{k,i}/\gamma_{k,i}^2 \tag{7}$$

where $e_{k,i}$ is a unit vector in the direction from the i-th body to the k-th body and $\gamma_{k,i}$ is the Euclidean distance between these two bodies.

Equation (6) defines a second order differential system. Assuming that the system evolves in a plane, then each of the variables $x_i(t)$ has two components, hence the system contains $2m+2$ functions. The system unique solution is determined giving initial conditions for each body, i.e. position and velocity at time $t=0$, which are those provided by the solved equilibrium positions.

The numerical solution of the system has been obtained using a standard code P.P. for m-body dynamical systems, based upon the Bulirsch-Stoer algorithm, which uses a varying time stepsize. The system was transformed into a first order system with twice the number of variables, as is standard practice. The so called softening parameter of the code was set to $\sigma = 0.00025$. Energy and momentum were conserved within a relative error of less than 10^{-10}. The system alignement was considered to have been lost as soon as the angle between the vector Earth-Mars and Venus-Saturn in Table 3 or Sun-Earth and Sun-Saturn in Table 4 exceeded 10%.

The system was solved for several values of α in the range 0.1-1.9. The times T in months for the loss of alignement (duration of the alignement) are given in the following Tables,

Table 3

α	0.1	0.3	0.5	0.7	1.	1.1	1.3	1.5	1.7	1.9
T	0.128	0.673	1.61	2.869	1.19	1.1	1.02	0.99	0.97	0.96

Table 4

α	0.1	0.3	0.5	0.7	0.8	0.9	1.1	1.3	1.5	1.9
T	0.138	0.73	1.72	3.01	2.0	1.36	1.1	1.02	0.99	0.96

In the Appendix A we give graphs of $T = T(\alpha)$ obtained by standard interpolation of the data given in the Table 3. Appendix B gives similar graphs obtained by changing the mass of Saturn (the higher curve corresponds to a 10% decrease, the two lower curves to increase of respectively 10% band 30%.

An inspection of the Tables and curves suggests the following conclusions:

1. For small α the alignement is lost in a matter of days, as naturally expected since small α imply a much higher orbital velocity. The duration of the alignement increases with α to a maximum of about 3 months at α about 0.8, the maximizing value of α depending on Saturn mass and increasing rather fast with decreasing mass of Saturn. Then T decreases rather fast and after going through an apparent secondary (probably a spurious feature, an artifact of the graphic procedure) small maximum appears to settle to a plateau of about one month.

2. Even the maximum computed values of α are clearly too small to validate a significant duration of the polar configuration, whose expected stability should extend at least over several thousand years

3. The dependence of T from α is clearly nonlinear and somewhat intriguing. Since only a limited number of discrete values of α have been considered in the numerical study of the evolution of the system, and since certain dynamical systems are known to present completely different and unexpected behaviour in even very small region of their parameter space (defined by α and the masses), we think that it is *impossible a priori* to state the instability of the system for all values of α and for acceptable changes in the planetary masses that may have resulted from a catastrophical collapse of the system. However we consider the existence of untested values of the parameters that make the system stable to be very unlikely.

It should also be noted that the system, while losing the alignement rather soon, remains a gravitationally tied system with a complex evolution.

4. Conclusions

The dynamical analysis performed on the aligned system consisting of Sun, Earth, Mars, Venus and Saturn in this order shows that alignement is lost rather fast, the maximum duration corresponding to only about 3 months when the distance Earth to Sun is about 120.000.000 km., i.e., about 20% less than presently. The dynamical analysis has been performed ignoring tidal effects and nongravitational (i.e. electromagnetic) forces, whose importance has been claimed by some authors, e.g. Ginenthal (The electrogravitic theory of celestial motion and cosmology, The Velikovskian IV, 3, 1999). Whether the introduction of such other forces may result in stabilizing the system is a question for future research.

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